arXiv:1109.4761v2 [physics.gen-ph] 6 Mar 2012

Towards $E_8 \times E_8'$ Unification

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Abstract. We present a framework having the potential to unify the fundamental interactions in nature by introducing new degrees of freedom. An attempt is made to explain the hierarchy between the weak scale and the coupling unification scale, which is found to lie close to Planck energies. A novel process leading to the emergence of symmetry is proposed, which not only reduces the arbitrariness of the scenario proposed but is also followed by significant cosmological implications. Phenomenology is in principle consistent with deviations from the Standard Model reported by LEP and Fermilab and includes the probability of detection of mirror fermions at the LHC.

PACS numbers: 14.80.Tt, 12.60.Nz, 12.60.Rc, 12.10.Dm, 12.10.Kt, 04.60.Nc, 89.75.Fb

1. Motivation

Attempts to unify fundamental interactions within a unique theory are not only based on aesthetic and philosophical grounds, but are also reinforced by experiments implying a convergence of the gauge-coupling strengths [1]. We built here upon previous work [2][3] trying to explain fermion family structure and the hierarchy between the electroweak (EW) symmetry scale and the coupling unification scale by introducing new fermions which we call katoptrons (from the Greek word for "mirror"), since it is imperative to distinguish them from ordinary mirror fermions appearing in alternative models. Stabilization of the EW scale is achieved by using a new gauge interaction $SU(3)_K$ becoming strong around 1 TeV. The Higgs mechanism is based on fermion condensates, similarly to technicolour [4][5]. If katoptrons did not carry $SU(3)_K$ charges, they would be the mirror partners of the known fermions.

It is assumed that the $SU(3)_K$ coupling is equal to the other couplings at the unification scale. One then finds that this coupling becomes strong around the weak scale, and the $SU(3)_K$ interactions are thus taken to be responsible for the breaking of the EW symmetry. Adding so many new particles seems to be required from a theory involving neither new elementary zero-spin particles, nor new energy scales fixed *ad hoc*, nor too many arbitrary parameters. Moreover, this approach can in principle produce testable predictions since the effects of these particles, if they exist at all, are expected to be studied at the LHC [6].

In order to bring these considerations a step closer to gravity, the initial gauge symmetry was previously enlarged to $E_8 \times E_8'$ in 10 spacetime dimensions, with known fermions contained in E_8 and katoptrons in E_8' [7]. Since stabilization of extra dimensions is hard, it makes sense to wonder if too many, and hard to detect, degrees of freedom were added in that work. Enlarging the gauge symmetry to such an extent could possibly suffice to bring us closer to gravity, avoiding the addition of extra dimensions by embedding the Lorentz group within a larger symmetry [8]. We proceed here towards this direction by adopting a bottom-up approach, going from lower energies where theories are well tested to higher energies where powerful theorems have to be surmounted, new theoretical methods have to be developed and experimental data are lacking.

2. Coupling unification, symmetry breaking and spinor gravity

One of the results of [2] is coupling unification at high energies, including the $SU(3)_K$ coupling. The starting point here is a different initial symmetry G breaking at energies Λ_{GUT} down to $SU(5)\times U(1)_X \times SU(5)' \times U(1)'_X \times SU(3)_K$. We investigate the running of the couplings in this case to see if the energy scales involved have theoretically and phenomenologically acceptable values. Under $SU(5)\times U(1)_X \times SU(5)' \times U(1)'_X \times SU(3)_K$, left-handed ordinary fermions F and right-handed katoptrons K are taken to transform as follows:

$$\begin{aligned} F_L^a &= (\bar{\mathbf{5}},-3,\mathbf{1},0,\mathbf{1})^a \oplus (\mathbf{10},1,\mathbf{1},0,\mathbf{1})^a \oplus (\mathbf{1},5,\mathbf{1},0,\mathbf{1})^a \\ K_R &= (\mathbf{1},0,\bar{\mathbf{5}},-3,\mathbf{3}) \ \oplus (\mathbf{1},0,\mathbf{10},1,\mathbf{3}) \ \oplus (\mathbf{1},0,\mathbf{1},5,\mathbf{3}) \end{aligned} \qquad (1)$$

where $a = 1,2,3$ is a fermion generation superscript, **1** and 0 denote non-abelian and abelian group singlets respectively, ordinary fermions are singlets under $SU(5)' \times U(1)_X' \times SU(3)_K$ and appear in 3 generations, while katoptrons are singlets under $SU(5) \times U(1)_X$ and triplets under $SU(3)_K$. In an assignment inspired by "flipped $SU(5)$" models [9], the $\bar{\mathbf{5}}$ of $SU(5)$ contains a lepton doublet and up-type antiquarks, the **10** a quark doublet, down-type antiquarks and a neutrino, and the $SU(5)$ singlet is a positively-charged lepton (positron etc.), and similarly for the katoptrons in $SU(5)'$.

The symmetry $SU(5) \times U(1)_X \times SU(5)' \times U(1)_X'$ breaks at energies Λ_{23} down to the Standard Model (SM) group times an abelian $U(1)_1'$ felt only by katoptrons. One is then left with $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_1' \times SU(3)_K$, where $U(1)_1$ is the hypercharge group with a rescaled coupling. The fields leading to this breaking are neglected in the 1-loop calculation of coupling renormalization below. While katoptrons interact with the same $SU(3)_C \times SU(2)_L$ interaction as known fermions at energies below Λ_{23}, they carry their own $U(1)_1'$ interaction down to the EW symmetry scale. Moreover, $SU(3)_K$ becomes strong near Λ_K, breaking itself and the EW symmetry [3].

The renormalization of the gauge couplings g_N at energy scales p for N_f fermions in the fundamental representation (rep) of $SU(N)$ at 1-loop is given by $\alpha_N^{-1}(p) = \alpha_N^{-1}(p_o) + c(N,N_f)\ln(p/p_o)$, with p_o some reference scale, $a_N = g_N^2/4\pi$ and $c(N,N_f) = (11N - 2N_f)/6\pi$. The katoptron coupling α_K evolves at scales ranging from Λ_K to the unification scale Λ_{GUT} according to $c_K \equiv c(3,8) = 17/6\pi$. The $SU(2)_L$ coupling α_2 and the $SU(3)_C$ coupling α_3 evolve according to $\tilde{c}_N \equiv c(N,12) = (11N-24)/6\pi$ at energies where both ordinary and katoptron fermions contribute to the beta functions, *i.e.* between Λ_K and Λ_{23}. Either below Λ_K, where katoptrons are massive and decouple, or when fermions and katoptrons interact with distinct groups, as is the case for all the $U(1)$ and the $SU(5)$, $SU(5)'$ couplings, couplings evolve according to $c_N \equiv c(N,6) = (11N-12)/6\pi$, with $N = 0$ for the α_X, α_X', α_1, α_1' couplings of $U(1)_X$, $U(1)_X'$, $U(1)_1$, $U(1)_1'$ respectively, and $N = 2,3,5$ for the $SU(2)_L, SU(3)_C$ couplings and the α_5, α_5' couplings of $SU(5)$, $SU(5)'$ respectively. The relevant boundary conditions, noting that $\alpha_K(\Lambda_K) \sim 1$ and $M_Z \sim 91.2$ GeV, are:

$$\begin{aligned}
\alpha(\Lambda_{GUT}) &\equiv \alpha_X(\Lambda_{GUT}) = \alpha_5(\Lambda_{GUT}) = \alpha_X'(\Lambda_{GUT}) = \alpha_5'(\Lambda_{GUT}) = \alpha_K(\Lambda_{GUT}) \\
\alpha_2(\Lambda_{23}) &= \alpha_3(\Lambda_{23}) \qquad \text{(Unification condition for } \mathrm{SU(3)_C \times SU(2)_L}) \\
\alpha_X^{-1}(\Lambda_{23}) &= \frac{25}{24}\alpha_1^{-1}(\Lambda_{23}) - \frac{1}{24}\alpha_5^{-1}(\Lambda_{23}) \text{ (Flipped SU(5) matching condition)} \\
(\alpha_X')^{-1}(\Lambda_{23}) &= \frac{25}{24}(\alpha_1')^{-1}(\Lambda_{23}) - \frac{1}{24}(\alpha_5')^{-1}(\Lambda_{23}) \text{ (Flipped SU(5)}' \text{ matching condition)} \\
\alpha_1^{-1} &\equiv \frac{3}{5}\alpha_Y^{-1}, (\alpha_1')^{-1} \equiv \frac{3}{5}(\alpha_Y')^{-1} \text{(Hypercharge normalization)} \\
\alpha_1^{-1}(Mz) &= 59.5, \alpha_2^{-1}(Mz) = 29.8, \alpha_3^{-1}(Mz) = 8.5 \text{ (Experimental input)}
\end{aligned} \tag{2}$$

These relations yield Λ_{GUT}, Λ_{23}, and Λ_K, assuming a big desert between these scales and decoupling of the heavier degrees of freedom [10]. Defining

$$A = 1 - \frac{c_3 - c_2}{\tilde{c}_3 - \tilde{c}_2}, \qquad B = \frac{\alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z)}{\tilde{c}_3 - \tilde{c}_2},$$

$$C = \frac{c_2 + (1-A)(c_5 - \tilde{c}_2)}{c_K - c_5}, \qquad D = \frac{\alpha_2^{-1}(M_Z) - B(c_5 - \tilde{c}_2) - 1}{c_K - c_5},$$
$$E = -\frac{D(c_K - c_0) - \frac{1}{24}\Big(B(c_0 - \tilde{c}_2) + 25\alpha_1^{-1}(M_Z) - \alpha_2^{-1}(M_Z)\Big) + 1}{C(c_K - c_0) - \frac{1}{24}\Big(A(c_0 - \tilde{c}_2) + \tilde{c}_2 - c_2\Big) - c_0} \tag{3}$$

and using the boundary conditions listed above, we find

$$\begin{aligned}\Lambda_{GUT} &= M_Z \exp\Big(E(1+C) + D\Big) = M_Z \times 10^{17} \sim 10^{19}\ \text{GeV} \sim \text{M}_{\text{Planck}}\\ \Lambda_{23} &= M_Z \exp\Big(AE + B\Big) = M_Z \times 6 \times 10^{15} \sim 5 \times 10^{17}\ \text{GeV, and}\\ \Lambda_K &= M_Z \exp E = M_Z \times 11 \sim 1\ \text{TeV}\end{aligned} \tag{4}$$

where $\Lambda_{23} = M_Z \exp B$ due to $A = 0$. Moreover, we find $\alpha(\Lambda_{GUT}) \sim 0.029$ and $\alpha_3(\Lambda_{23}) \sim 0.036$. In addition, one observes that

$$\Lambda_K \sim M_{\text{Planck}} \exp\Big(-\frac{6\pi}{17\alpha(\Lambda_{GUT})}\Big), \tag{5}$$

a relation rendering transparent the dynamical solution of the hierarchy problem due to katoptrons. The value of a single parameter remains then to be justified more fundamentally, *i.e.* $\alpha(\Lambda_{GUT})$, if a relation of the form $\Lambda_{23} \sim M_{\text{Planck}}\ \alpha_3(\Lambda_{23})$ can be produced from the dynamics. Furthermore, the value of Λ_{23} renders this scenario safe, at first glance, with regards to proton decay.

It has to be emphasized that if katoptrons kept interacting with the same hypercharge interaction as ordinary fermions, coupling unification would be impossible due to the faster running of the $U(1)_1$ coupling, as is obvious by the slopes of Figure 1. This problem was obviated in [2] by the appearance of a relatively low scale of Pati-Salam symmetry breaking, something not possible here. In addition, inspection of Figure 1 shows that had katoptrons kept their own distinct $SU(3)_C'$ and $SU(2)_L'$ interactions, coupling unification would require an EW symmetry breaking scale Λ_K that would be too low. Furthermore, in such a case the scale Λ_{23} would be too low to avoid an observable proton-decay rate. On the other hand, adding artificially additional matter in the present theory to slow down the running of the $SU(3)_C$ and $SU(2)_L$ couplings to enable them to unify with an hypothetical common $U(1)_Y$ coupling would require Λ_K to be too high for EW symmetry breaking. The value of Λ_{GUT} is close to M_{Planck} (see Figure 1), a result due to the slower running of some of the gauge couplings, since above Λ_K both fermions and katoptrons contribute to the renormalization of the $SU(2)_L$ and $SU(3)_C$ couplings. This is also due to the use of the $SU(5) \times U(1)_X$ group and its primed partner. It is not expected that these results change significantly by higher-order calculations or by including contributions of the fields responsible for symmetry breaking.

Next, the symmetry breaking chain from a group G down to $SU(5) \times U(1)_X \times SU(5)' \times U(1)_X' \times SU(3)_K$ and then down to the SM is explored using [11]. Non-zero vacuum expectation values (vevs) of effective composite fields are taken to lead to the breaking channels needed. These are fermion condensates arising non-perturbatively to safeguard gauge invariance at tree level. Coupling unification forces us to keep a distinct $U(1)_1'$ for katoptrons breaking at Λ_K in a way consistent with EW radiative corrections. Therefore,

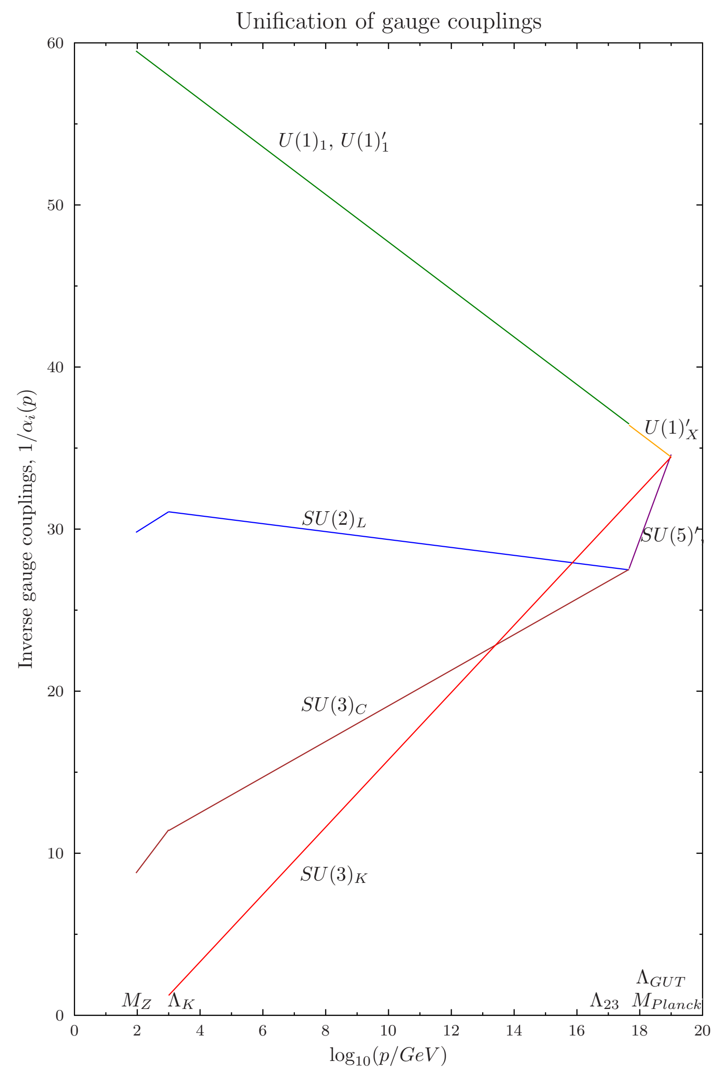

Figure 1. The renormalization of the gauge couplings at one loop.

apart from fermion condensates of the form $<\bar{K}K>$, gauge-invariant 4-fermion condensates $<\bar{K}K\bar{K}F>$ are assumed to break the two hypercharge symmetries down to their diagonal subgroup $U(1)_Y$ and then together with $SU(2)_L$ down to $U(1)_{em}$ near the weak scale. Such operators arise again non-perturbatively and are also needed to "feed" mass to fermions [3].

In order to have one generation of fermions in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ of $SU(5)$, in addition to the right-handed neutrino, needed for the SM particles to fit inside $SU(6)$ in an anomaly-free way (and similarly for katoptrons in $SU(6)'$), we assign fermions in $\bar{\mathbf{6}} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$ [12]. One generation of fermions fits inside 2 copies of $\bar{\mathbf{6}} = (\bar{\mathbf{5}}, 1) \oplus (\mathbf{1}, -5)$ and 1 copy of $\mathbf{15} = (\mathbf{5}, -2) \oplus (\mathbf{10}, 1)$. A $\bar{\mathbf{5}}$ pairs up with a $\mathbf{5}$ under $SU(5)$ acquiring thus GUT-scale masses. We are thus left with $(\bar{\mathbf{5}}, 1) \oplus (\mathbf{1}, -5) \oplus (\mathbf{10}, 1)$ under $SU(5) \times U(1)_X$, i.e. a full fermion generation, plus a neutral lepton for each generation, and similarly for katoptrons. Assuming that effective fields in the $\mathbf{35}$ of $SU(6)$ and $SU(6)'$ acquire non-zero vevs at Λ_{GUT}, $SU(6) \times SU(6)'$ breaks down to $SU(5) \times U(1)_X \times SU(5)' \times U(1)_X'$. Note that this is not the only possible breaking channel for each $SU(6)$ left invariant by $\mathbf{35}$, $SU(4) \times SU(2) \times U(1)$ and $SU(3) \times SU(3) \times U(1)$ being other examples. These are however phenomenologically unacceptable, since in these cases quarks would not carry $SU(2)_L$ charges. Therefore, it has to be proven that the "flipped" $SU(5)$ is the preferred symmetry-breaking channel.

With regards to the breaking of $SU(5) \times U(1)_X \times SU(5)' \times U(1)_X'$ down to $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_1'$ at Λ_{23}, since katoptrons interact with the same $SU(3)_C$ and $SU(2)_L$ as ordinary fermions, we use the fact that the $\mathbf{35}$ of $SU(6)$ contains a $(\mathbf{24}, 0)$ under $SU(5) \times U(1)_X$, which in its turn contains $(\mathbf{1{,}1}, 0) \oplus (\mathbf{8{,}1}, 0) \oplus (\mathbf{1{,}3}, 0)$ under $SU(3) \times SU(2)_L \times U(1)_{\tilde{1}} \subset SU(5)$ (and similarly for the primed groups). Assuming that a 4-fermion operator with quantum numbers $(\mathbf{24{,}24})$ under $SU(5) \times SU(5)'$ acquires a non-zero vev near Λ_{23}, it breaks the $SU(3)_C \times SU(2)_L \times U(1)_{\tilde{1}}$ subgroups of each of the $SU(5)$ and $SU(5)'$ to their diagonal subgroup, while $U(1)_X$, $U(1)_X'$ are left intact. This can be achieved assuming that not only the singlet under $SU(3)_C \times SU(2)_L \times SU(3)_C' \times SU(2)_L'$ but also the $(\mathbf{8{,}1{,}8{,}1})$ and $(\mathbf{1{,}3{,}1{,}3})$ effective fields acquire non-zero vevs. This is a step requiring extension of usual most-attractive channel (MAC) arguments, even though both of the corresponding channels are attractive. The fact that a 4-fermion operator is needed for this breaking might explain the relation $\Lambda_{23} \sim \alpha_3(\Lambda_{23})\Lambda_{GUT}$ conjectured above, since it would correspond to a higher-order effect. Further breaking to the SM takes place by additional effective fields transforming as a $\mathbf{20}$ of $SU(6)$, and similarly for $SU(6)'$, which decomposes under $SU(5) \times U(1)_X$ like $\mathbf{20} = (\mathbf{10}, 1) \oplus (\mathbf{10}, -1)$. Near the $SU(5) \times SU(5)'$ breaking scale Λ_{23}, these fields are taken to acquire non-zero vevs and also break $SU(5) \times U(1)_X \times SU(5)' \times U(1)_X'$ down to $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_1'$, mixing their $U(1)_X$, $U(1)_X'$ charges with the charges of the $U(1)_{\tilde{1}}$ and $U(1)_{\tilde{1}}'$ groups in the final hypercharge groups $U(1)_1$, $U(1)_1'$.

Note that the $\mathbf{20}$ can also leave invariant the $SU(3) \times SU(3)$ subgroup of $SU(6)$, so in order to achieve the desired breaking channel, it is imperative not only to use the fields in the $\mathbf{35}$ as described above, but to also to assume that the latter are the ones determining the dominant breaking channel direction. We will see shortly how the effective fields $\mathbf{20}$ and $\mathbf{35}$ in each of $SU(6)$ and $SU(6)'$ might emerge naturally in this theory. Moreover, one has to demonstrate that the alternative breakings of $SU(5)$ to $SU(4) \times U(1)$, and similarly for their

primed partners, is disfavoured. The reason why alternative embeddings of the $SU(5)\times U(1)_X$ groups into $SO(10)$ or a Pati-Salam symmetry for instance are not considered will become clear in the following.

Our next goal is to expand the larger symmetry $SU(6)\times SU(6)'\times SU(3)_K$ at the energy scale Λ_{GUT} further. Apart from being responsible for the dynamical EW symmetry breaking, $SU(3)_K$ plays the role of a gauged generation group for katoptrons. In addition, it prohibits gauge invariant vector-like masses corresponding to fermion bilinear operators formed by combining the known fermions with katoptrons. Even though it would be nice to have a generation gauge symmetry for the known fermions as well, this is excluded by phenomenological reasons, i.e. by the absence of flavour-changing neutral currents. It is however reasonable to assume, for naturalness and symmetry reasons, that such a generation symmetry $SU(3)_F$ existed at higher energies only to be broken at lower energies. An initial symmetry group of the form $SU(6)\times SU(6)'\times SU(3)_F\times SU(3)_K$ would nonetheless have rank 14. In principle, such a symmetry may be embedded within a larger symmetry to allow for unification.

For that purpose, we consider $G = E_8 \times E_8'$. Known fermions fit in $SU(6)\times SU(3)_F$ embedded in $E_7 \subset E_8$, while katoptrons fit in $SU(6)'\times SU(3)_K$ embedded in $E_7' \subset E_8'$. Fermions sit in the **248** reps of the two E_8s, and the ones in the E_8' exhibit the opposite chirality from the ones in E_8. We further assume that $SU(3)_F$ breaks at Λ_{GUT}. The **56** and **133** of E_7 transform under $SU(6)\times SU(3)$ like $\mathbf{56} = (\overline{\mathbf{6}},\overline{\mathbf{3}})\oplus(\mathbf{6},\mathbf{3})\oplus(\mathbf{20},\mathbf{1})$ and $\mathbf{133} = (\mathbf{15},\overline{\mathbf{3}})\oplus(\overline{\mathbf{15}},\mathbf{3})\oplus(\mathbf{35},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$. If fermions initially sit in these reps, vector-like particles contained in $(\mathbf{20},\mathbf{1}),(\mathbf{35},\mathbf{1})$ and $(\mathbf{1},\mathbf{8})$ obtain GUT-scale masses. We then need 2 copies of fermions in the **56** of E_7 to give us the 2 $\overline{\mathbf{6}}$s, and 1 copy in the **133** of E_7 containing the **15** of $SU(6)$ described above. This content, replicated thrice, corresponds to the fermion generations. Similar considerations apply for the primed groups corresponding to katoptrons. We discuss later fermions transforming under the conjugate $(\mathbf{6},\mathbf{3})$ and $(\overline{\mathbf{15}},\mathbf{3})$.

How do E_7 and E_7' break down to $SU(6)\times SU(3)_F$ and $SU(6)'\times SU(3)_K$? The **133** of E_7 contains the $(\mathbf{35},\mathbf{1})$ of $SU(6)\times SU(3)$ which is needed for the breaking of $SU(6)\times SU(6)'$ down to $SU(3)_C\times SU(2)_L\times U(1)_{\tilde{1}}\times U(1)_{\tilde{1}}'\times U(1)_X\times U(1)_X'$. Similarly, the **56** of E_7, when decomposed under $SU(6)\times SU(3)$, contains the $(\mathbf{20},\mathbf{1})$ needed for the breaking of $U(1)_{\tilde{1}}\times U(1)_{\tilde{1}}'\times U(1)_X\times U(1)_X'$ down to $U(1)_1\times U(1)'$. Considering non-zero vevs of effective fields in the **56** and **133** of E_7 and E_7' may lead to the breaking sequence needed, since not only do these break E_7, E_7', but they also contain the $(\mathbf{20},\mathbf{1})$ and $(\mathbf{35},\mathbf{1})$ fields for the breaking of the symmetries to the SM. The embedding of $E_7\times E_7'$ in $E_8\times E_8'$ is now straightforward, with left-handed and right-handed fermions initially contained in the fundamental reps of E_8 and E_8' respectively. Noting that $E_7\times SU(2)\subset E_8$, and that the **248** of E_8 decomposes under $E_7\times SU(2)$ like $\mathbf{248} = (\mathbf{133},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})\oplus(\mathbf{56},\mathbf{2})$, we find 2 copies of fermions in the **56** and 1 copy of the **133** in each of the E_8 and E_8', assuming that the extra $SU(2)$, $SU(2)'$ symmetries in both E_8 and E_8' are also broken.

Regarding the breaking of E_8 and E_8' down to $E_7\times SU(2)$ and its primed copy, the symmetric tensor product of 2 fundamental reps of E_8 yields $\mathbf{248}\times\mathbf{248} = \mathbf{1}\oplus\mathbf{3875}\oplus\mathbf{27000}$. In fact, $E_7\times SU(2)$ is the only symmetric subgroup of E_8, denoted sometimes in this context as

$E_{8(-24)}$, left invariant by **3875**. If fields, like fermion bilinears, having such quantum numbers acquired non-zero vevs, they could provide us with the desired symmetry-breaking channel for E_8 and E_8', assuming that breaking to a symmetric subgroup is preferred over other channels. Moreover, the **3875** of E_8 contains the **56** and **133** effective fields under E_7 that could lead to the symmetry breaking channels exposed above. The **3875** leaves $E_7\times SU(2)$ invariant, but it is quite large, and it being formed is hard to justify by MAC arguments. The **248** contains the **56** and **133** of E_7 as well, and the corresponding channel is also attractive [13], so having a non-zero vev of an effective field in this rep might also suit our purposes. Since adding fundamental scalars is avoided, a candidate for this field is formed by an antisymmetric tensor product of two fermions in **248**, i.e. $\mathbf{248}\times\mathbf{248}=\mathbf{248}_a$, where the subscript a refers to its antisymmetric nature. This field however is not a Lorentz scalar, a fact used later in this work.

Next, we inquire how the generation symmetry of the ordinary fermions $SU(3)_F$ breaks. We relax one of our assumptions and take the coupling g_F of E_7 to be much larger at Λ_{GUT} than the coupling g_K of E_7', something lying at the heart of parity asymmetry. The large value of g_F is assumed to trigger the self-breaking of $SU(3)_F$ down to $SU(2)_F$, via a non-zero fermion bilinear vev transforming under the $\bar{\mathbf{3}}$ of $SU(3)_F$ originating from $\mathbf{3}\times\mathbf{3}\to\bar{\mathbf{3}}\oplus\mathbf{6}$. This is further broken down to $U(1)_F$ with a vev transforming under the adjoint of $SU(2)_F$ coming from $\mathbf{2}\times\mathbf{2}\to\mathbf{1}\oplus\mathbf{3}$, assuming that the singlet of this channel does not determine the correct vacuum. The remaining $U(1)_F$ breaks when the fundamental reps of $SU(6)$ pair up with each other, with the **6** of **133** pairing up with the $\bar{\mathbf{6}}$ of **56** of E_7. This scenario is in principle consistent with the self-breaking of the mirror-fermion generation symmetry $SU(3)_K$ at $\Lambda_K\sim 1$ TeV, since both symmetries self-break when their gauge couplings become strong [3]. To assure that this does not create problems with coupling unification, we use the relation $g(\Lambda_{23})=\frac{g_F g_K}{\sqrt{g_F^2+g_K^2}}\sim g_K(\Lambda_{23})$ holding for the (α_5, α_5') and (α_X, α_X') couplings due to the breaking of their respective gauge groups at Λ_{23} down to their diagonal subgroups, since $g_F >> g_K$. All fermions at Λ_{23} are then left with common abelian and non-abelian couplings, *i.e.* the weakest ones.

To sum up, denoting as LG the symmetry $SU(2)\times SU(2)'$, the following symmetry breaking chain is obtained, starting from $M_{\text{Planck}}\sim\Lambda_{GUT}$:

$$\begin{array}{lcc}
E_8\times E_8' & (\text{at }\Lambda_{\text{GUT}}) & \rightarrow\\
E_7\times SU(2)\times E_7'\times SU(2)' & \text{''} & \rightarrow\\
SU(6)\times SU(3)_F\times SU(6)'\times SU(3)_K\times[SU(2)\times SU(2)'] & \text{''} & \rightarrow\\
SU(5)\times U(1)_X\times SU(5)'\times U(1)_X'\times SU(3)_K\times LG & (\text{at }\Lambda_{23}) & \rightarrow\\
SU(3)_C\times SU(2)_L\times U(1)_1\times U(1)_1'\times SU(3)_K\times LG & (\text{at }\Lambda_{\text{K}}) & \rightarrow\\
SU(3)_C\times U(1)_{em}\times LG & &
\end{array} \tag{6}$$

We write down the broken LG and its $SU(2)$ components, as well as E_7, E_7', $SU(3)_F$, $SU(6)$ and $SU(6)'$ in order to render the breaking sequence more transparent.

Next, we address the appearance of conjugate generations in the considerations above. These are sometimes referred to as mirror families, and would in principle appear within each of the E_8, E_8'. However, we assume that mirror fermions are just coming from a second E_8,

i.e. E_8'. What happens to the conjugate generations within each of the E_8, E_8'? This issue first appeared when we stepped from $SU(6)$ to E_7. If these conjugate generations were the mirror partners of the rest of the fermions, i.e. related to each other via a parity transformation interchanging their chirality, they would probably pair up with them, acquiring unification-scale gauge-invariant vector-like masses and disappearing from the low energy spectrum. If there were a symmetry reason why this is forbidden, they would anyway never have masses much above 1 TeV, since in that case they would raise the weak scale at unacceptable levels. But even in that case, they would either share a gauged generation symmetry with the known fermions, something which is experimentally excluded, or not have such a gauge symmetry, in which case our dynamical symmetry breaking scenario involving strongly-interacting mirror fermions would break down. We must therefore dispense of the conjugate copies within each of the E_8, E_8' in a consistent manner. This problem was circumvented in [7] by introducing extra dimensions, since in 10 dimensions one may define Majorana-Weyl spinors and impose a chirality condition, identifying thus the two kinds of fermion generations. In the present 4-dimensional approach however this trick is inapplicable.

The answer lies within the channel the groups E_8 and E_8' break [11]. The group E_8 has just 2 symmetric subgroups, $SO(16)$ and $E_7 \times SU(2)$. Charge conjugation and parity transformations on fermion reps in each of these subgroups are distinct, flipping the sign of a different number of E_8 roots. When embedding $SO(16)$ in E_8, charge conjugation C' amounts to complex conjugation, i.e. for a fermion rep $\mathbf{R}$ in $SU(6) \subset SO(16)$ for instance, $C' : \mathbf{R} \to \overline{\mathbf{R}}$ and $\mathbf{6}_L \to \overline{\mathbf{6}}_L$, the bar denoting complex conjugation, while parity transformation P' changes chirality with no complex conjugation, *i.e.* $P' : \mathbf{R} \to \mathbf{R}$ and $\mathbf{6}_L \to \mathbf{6}_R$. Consequently, $C'P'$ transformations taking particles to their antiparticles are distinct from parity transformations. On the contrary, in the embedding of $SU(6) \subset E_7$ in E_8, charge conjugation C leaves the fermion rep invariant, *i.e.* $C : \mathbf{R} \to \mathbf{R}$ and $\mathbf{6}_L \to \mathbf{6}_L$, while a parity transformation P implies not only chirality change but charge conjugation as well, *i.e.* $P : \mathbf{R} \to \overline{\mathbf{R}}$ and $\mathbf{6}_L \to \overline{\mathbf{6}}_R$, so that $CP : \mathbf{6}_L \to \overline{\mathbf{6}}_R$. Parity and CP transformations have an identical effect in the $E_7 \times SU(2)$ case. Consequently, conjugate families appearing in E_7 and E_8 here are taken to be the antiparticles of fermions, not their mirror partners. The fact that these are not observed in nature characterizes the "baryon asymmetry of the Universe" and is a distinct problem from the one of "mirror" fermions. The reason for choosing this particular symmetry-breaking channel is now clear, since a breaking involving $SO(10) \subset SO(16)$ leading to a Pati-Salam (PS) scenario creates problems with conjugate generations. An alternative PS scenario could in principle still be recovered via a channel involving E_7 and E_6 [7], but it leaves no room for the symmetry LG, which is central to the discussion below.

To proceed, we argue that $SU(2) \times SU(2)' \approx LG$ contains the Lorentz symmetry $SO(3,1)$, an assumption motivated by the proximity of Λ_{GUT} to M_{Planck}. With regards to the non-compact nature of the Lorentz symmetry, note that we are dealing with the complexified versions (C) of the corresponding groups. Therefore, to be more explicit, one should write the E_8 decomposition above as $E_7(C) \times SL(2,C) \subset E_8(C)$ [14]. One then gets from the two E_8s the group $SO(4,C) \approx SU(2,C) \times SU(2,C)'$ (since $SU(2,C) \approx SL(2,C)$) which has both $SO(4)$ and $SO(3,1)$ as subgroups. A relevant mechanism should obviously be provided in

order to lead to the relevant symmetry breaking and to the observed signature in nature. The LG group is therefore assumed to contain the Lorentz group and it is taken to be a global, not a local symmetry. We list some arguments supporting this assumption. First, spin-1 gauge bosons associated with a gauged LG are neither observed nor expected. Second, there are no prohibitive phenomenological constraints enforcing a local Lorentz symmetry [15]. Third, taking Lorentz symmetry to be a global symmetry spontaneously broken allows the identification of some of the corresponding Goldstone bosons with the usual gravitons [8][16]. Fourth, considering LG as an unbroken symmetry would not allow the pairing of the fermions sitting in the **6** and $\bar{\mathbf{6}}$ of $SU(6)$ needed to obtain the required fermion content. Last, breaking the Lorentz symmetry via an antisymmetric fermion tensor product transforming like $\mathbf{248}_a$ under E_8 is needed in order to obtain the symmetry-breaking channel to the SM, as described below. In order to render the relation between LG and the Lorentz group consistent, one needs to equate the $SO(3,1)$ coupling with the gravitational coupling. For coupling unification to work, one should extend the previous parity-breaking relation $g_F >> g_K$ from the two E_7 to the two E_8 groups. After the breaking of $SU(2,C) \times SU(2,C)'$ down to the Lorentz group by a mechanism discussed later, $SO(3,1)$ should be left with the g_K coupling of E_8', *i.e.* the weakest one.

Before continuing, we need to address the Coleman-Mandula theorem prohibiting the total symmetry from being a direct product of a local Lorentz group with a gauged symmetry [17]. We choose to follow [8], claiming that our starting point is a topological symmetric phase, in which the metric is initially absent and no S matrix is defined. As described below, the metric appears only as the product of non-zero vevs in the Higgs phase. When the metric appears, Lorentz symmetry is a global symmetry presenting no further problems.

What follows below is a rough, initial investigation on whether the picture just described could in principle be incorporated in models unifying the Lorentz with the gauge groups, without going deep into the intricacies of such approaches. Within a framework similar to the one in [8], we consider a metric $g_{\mu\nu} = E^m_\mu(x) E^n_\nu(x) \eta_{mn} = E^m_\mu(x) E_{\nu m}(x)$, where $\mu,\nu = 0,\ldots,d$ are spacetime indices, $m,n = 0,\ldots,d$ are indices corresponding to the internal Lorentz symmetry of a d-dimensional spacetime with $\eta_{mn} = \mathrm{diag}(-1,1,\ldots,1)$, and

$$\begin{array}{ll} E^m_\mu(x) = \,<\tilde{E}^m_\mu(x)>\sim \delta^m_\mu M_{\text{Planck}} & \text{for } \mu,\text{m} = 0,\ldots,3 \\ E^m_\mu(x) = \,<\tilde{E}^m_\mu(x)>\sim 0 & \text{for } \mu,\text{m} = 4,\ldots,\text{d} \end{array} \tag{7}$$

are soldering forms (vielbeins), i.e. vevs of operators $\tilde{E}^m_\mu(x)$ breaking the Lorentz symmetry spontaneously. The order parameter of this transition is a 1-form, not a 0-form. Global Lorentz symmetry is preserved only under combined Lorentz transformations on the internal Lorentz (m) and the ordinary spacetime (μ) indices. Symmetric fluctuations of such a metric around the Minkowski spacetime are expected to produce Goldstone bosons identified with gravitons [8] [15]. In the spinor gravity approach [15], these vevs have a dynamical origin since they are expressed as fermion bilinear operators:

$$\tilde{E}^m_\mu(x) = \frac{i}{2}\{\bar{\Psi}(x)\gamma^m\partial_\mu\Psi(x) - \partial_\mu\bar{\Psi}(x)\gamma^m\Psi(x)\} \tag{8}$$

where γ^m are Dirac matrices in d dimensions and $\Psi,\bar{\Psi}$ are Grassmann variables in the

irreducible spinor rep of the d-dimensional Lorentz group.

A relevant partition function, effective action and effective potential can then be formally defined, a prerequisite being an anomaly-free functional measure $\mathscr{D}\Psi$ preserving Lorentz and diffeomorphism invariance. This is expected to lead in principle, in lowest order in the effective potential expansion, to equations similar to the ones of General Relativity [15], in a way that spacetime is not treated as background but is incorporated in the equations non-perturbatively. In such a picture, physical distances are induced by fermion correlation functions and the appearance of a metric is inherently quantum-mechanical. Difficulties in quantizing gravity would show up in possible gravitational anomalies and in regularizing the corresponding effective potential. Although the finite number of counterterms in this context is encouraging for renormalizability, we do not pursue further the highly non-trivial issues arising in this setting.

The approaches quoted above use large orthogonal groups in which the embedding of the Lorentz group takes place, treating Lorentz and gauge transformations in a unified manner as rotations in a higher-dimensional space (14 total dimensions in [8] corresponding to $SO(3,11)$, 16 total dimensions in [15] corresponding to $SO(16)$). The interpretation of these extra dimensions differs slightly in the two approaches. In [15], gauge symmetry arises by compactification of extra space dimensions. Therefore, in a complete approach we have to replace ordinary derivatives by covariant ones in the quantities appearing in the four-dimensional S_{eff}, while this is not necessary for the higher-dimensional S_f. On the other hand, extra dimensions in [8] are considered as internal dimensions corresponding to a unifying orthogonal group. The two approaches are similar, with the action proposed in [8] corresponding to the 1-loop effective action of [15]. This correspondence is realized when spacetime derivatives in 4d appearing in [15] are replaced by gauge covariant derivatives, and fermion bilinears are treated as effective fields.

The problem with large orthogonal groups however, as we saw above, is the emergence of conjugate generations which cannot be considered as anti-generations, but are just mirror copies of ordinary fermions. To solve this problem and make connection with our model, we extend the unification symmetry to $E_8 \times E_8'$, adopting the fermion-bilinear approach for the soldering forms [15] in order to maintain the dynamical interpretation of the breaking of $G = E_8 \times E_8'$, where internal dimensions are connected with the appearance of gauge symmetries in 4d. We take some first steps exploring whether our model could give us a mechanism in principle compatible with such a dynamical metric-generation scenario. In fact, non-zero vevs of antisymmetric fermion bilinears sitting in the $\mathbf{248}_a$ of E_8 and E_8' might lead to the breaking sequence needed. The $SU(2)$ triplets in **(1,3)** and doublets in **(56,2)** contained in the decompositions of the $\mathbf{248}_a$ of each of the E_8, E_8' under $E_7 \times SU(2)$ can break spontaneously LG after acquiring non-zero vevs, leading thus to a dynamically generated metric tensor by condensates, assuming that $<\bar{\Psi}\gamma^m\partial_\mu\Psi>=0$ for $m>3$. The vevs of fields in the **(1,3)** of $E_7 \times SU(2)$ and $E_7' \times SU(2)'$ constitute the antisymmetric MAC of $E_8 \times E_8'$ breaking down to $E_7 \times E_7'$, possibly justifying their dominance in determining spacetime dimensionality over the **(56,2)** and **(133,1)** vevs breaking subsequently $E_7 \times E_7'$ and the remnant of LG.

In order for LG to break down to its diagonal subgroup $SO(3,1)$, one should really consider vevs of 4-fermion composite operators transforming as **(1,3,1,3)** under $E_7 \times SU(2) \times E_7' \times SU(2)'$. In such a case, the interpretation of the metric would come from a relation of the form $g_{\mu\nu} = <\tilde{E}^m_\mu \tilde{E}_{\nu m}> \sim <\tilde{E}^m_\mu><\tilde{E}_{\nu m}>$. Moreover, since non-zero vevs of 4-fermion operators already appeared in the previous section and were related to the breaking of the two $SU(5)$ symmetries down to the SM gauge group at energies on the order of Λ_{23}, it makes sense to inquire whether the 4-fermion operators related to the metric are formed around M_{Planck} (just where the E^m_μ are anyway expected to appear), or around Λ_{23} instead. Leaving this issue and related interesting phenomenological consequences to a future investigation, we note that a deeper analysis of this scenario might have the potential of not only explaining the dimensionality of our space-time, but its signature as well, i.e. why the group $SO(3,1)$ emerges instead of $SO(4)$ or $SO(2,2)$ for instance.

The next step is to consider the effective fermionic action in 4d in a derivative and fermion-field expansion to 1-loop taking a unified form similar to the one in [15]

$$S_{eff} \sim \int d^4x \det\left(E^m_\mu(x)\right)\left(c_1 + c_2 R + c_3 \bar{\Psi}(x)\gamma^m E^\mu_m(x) D_\mu \Psi(x) + \ldots\right) \tag{9}$$

plus gauge kinetic terms, where $\det\left(E^m_\mu(x)\right) = \sqrt{\det(-g_{\mu\nu})} \neq 0$ while $\mu,\nu,m = 0,\ldots,3$, $c_{1,2,3}$ are constants and D_μ is the gauge-covariant derivative corresponding to the $E_7 \times E_7'$ gauge symmetry after compactification. The first term gives a cosmological constant Λ, the second the Ricci curvature, and the third the action in [8]. The result above is expected to stem in principle from an action containing the expression

$$S_f \sim \int d^dx \det(\tilde{E}^m_\mu) \tag{10}$$

in d dimensions, while other invariants give rise to higher order terms. Details on the compactification mechanism shedding light on questions like "why are other dimensions left compactified after the $E_7 \times E_7'$ symmetry breaking?" are left for future studies.

Consider now a 2-form $\tilde{G}_{mn}$ as a generalized, internal "metric" defined on a d-dimensional manifold expressed as $\tilde{G}_{mn} = \tilde{E}^p_m \tilde{E}_{np}$, with m,n,p spanning all d dimensions and $\tilde{E}^p_m$ defined as before. Non-zero vevs sitting in the $\mathbf{248}_a$ of E_8 and E_8' give rise to a breaking of the initial symmetry down to $E_7 \times E_7'$ and to an effective metric $\tilde{G}_{eff}$. In lowest order, after splitting the coordinates to spacetime (x) and internal (y) ones, this should take the form

$$\tilde{G}_{eff} = \begin{pmatrix} g_{\mu\nu} + g_{kl} A^k_\mu A^l_\nu & g_{kl} A^k_\mu \omega^l_b \\ g_{kl} A^k_\nu \omega^l_a & g_{kl} \omega^k_a \omega^l_b \end{pmatrix} \tag{11}$$

where the $\mu,\nu = 0,\ldots,3$ indices refer to x- while the $a,b,k,l = 4,\ldots,d$ indices to y-coordinates. Assuming that lower-dimensional fields are independent of y and imposing local invariance of $\tilde{G}_{eff}$ under diffeomorphisms, we identify the 4d effective fields as $g_{\mu\nu}(x) = E^m_\mu(x) E_{\nu m}(x)$ (Riemannian metric), $A^a_\mu(x)$ (Spin-1 Kaluza-Klein $E_7 \times E_7'$ fields) and $g_{kl}(y)\omega^k_a \omega^l_b$ (Spin-0 fields, metric of internal dimensions). The Maurer-Cartan 1-forms on the internal coordinates y^b defined as $\omega^a \equiv \omega^a_b dy^b$ are dual to the Killing vectors $K_b \equiv K^a_b \partial_a$ satisfying the commutation relations $[K_a, K_b] = c^k_{ab} K_k$ with c^k_{ab} the $E_7 \times E_7'$ structure constants, i.e. $\omega^a K_b = \delta^a_b$. Obviously, Killing vectors preserve the metric of the internal dimensions under

the transformation of the coordinates $y^a \to y^a + \varepsilon^b(x)K_b^a$, with $\varepsilon^b(x)$ arbitrary functions of x. These transformations correspond to isometries of the internal manifold expressed by the $E_7 \times E_7'$ algebra. It is important to realise that in this setting, gauge fields are conceptually associated not only with isometries of an internal manifold, but also with appropriate fermion two-point correlation functions.

We then investigate the dimensionality d of space to integrate our Lagrangian over. Since spacetime and internal dimensions are treated on an equal footing, naturalness reasons lead us to consider the action as an integral over a manifold having the isometry $E_8 \times E_8'$. The proper number of (complex) dimensions to integrate our Lagrangian over is then $d = 16$, equal to the number of roots of the groups involved. Integrating over the 14 extra internal dimensions should leave us with 2 complex, i.e. 4 real, ordinary spacetime dimensions. The internal dimensions are then assumed to be compactified at a size of around $1/M_{\text{Planck}}$ to avoid the appearance of Kaluza-Klein excited states at energies lower than M_{Planck}. We do not study effects arising from 0-spin fields connected to the compactification process such as the dilaton. The properties of a 4d effective potential formed by such scalars associated with the "shape" of the compactification space might shed light on Λ, its relative size to the root norm $1/M_{\text{Planck}}$ and other cosmological issues like inflation. In any case, these are expected to decouple at lower energies since they have M_{Planck} masses.

It is conjectured below that the space with the isometries needed is the quotient space of the 16d maximal torus T^{16} by the lattice $\pi\Gamma_8 \times \pi\Gamma_8$ generated by the roots of $E_8 \times E_8'$ multiplied by integer multiples of π. Allowed coordinate transformations take the form $\delta y^a \in \pi\Gamma_8 \times \pi\Gamma_8$, *i.e.* $\delta y^a = \pi \sum_b m_b e_b^a$, with e_b a basis of the root space and m_b integers. Roots can be associated with the Killing vectors K_b giving rise to the metric isometries. The conserved conjugate momenta $p_b \sim iK_b$ corresponding to Killing vectors are associated with the same lattice and can be expressed as $p_a = \sum_b m_b' e_a^b$, where m_b' are again integers. Invariance requirements for the metric and single-valuedness of the plane wave $e^{i2p_b y^b}$ after coordinate transformations imply that $\sum_a p_a \delta y^a = \pi n, n \in \mathbf{Z}$, which requires that $p_a = \sum_b \tilde{m}_b \tilde{e}_{ba}$, with $\tilde{e}_b$ a basis of a space dual to e^b, *i.e.* $\sum_c e_a^c \tilde{e}_{cb} = \delta_{ab}$, and $\tilde{m}_b$ integers. Simultaneous validity of the expressions for the momenta above can only be achieved if the underlying lattice is self-dual, like the lattice $\Gamma_8 \times \Gamma_8$. Such lattices exist only in $d = 0 \pmod 8$ dimensions. When $d = 16$, the only such lattices are $\Gamma_8 \times \Gamma_8$ and Γ_{16}. We note that the same space has already been used in the different context of heterotic string theory [18]. After symmetry breaking, LG should carry traces of this discrete structure, while taking the root norm to be of size $1/M_{\text{Planck}}$ is expected to approximate satisfactorily the continuum at low energies. More work is obviously needed in order to check rigorously the validity of the construction just described.

In the following, and especially in the next section, an effort is made to motivate further our choice of $E_8 \times E_8'$ as a unification symmetry group, apart from the self-duality feature which might prove to be unique and crucial for unifying spacetime with gauge symmetries. The arguments presented below might reduce the arbitrary nature of such a choice. As will be shown later, the appearance of this symmetry might be due to a relevant phase transition and related to the fact that each of the two 8d E_8 lattices, which are also even and self-dual, offer the densest sphere-packing known in 8d. This property allows by the way their use

in other scientific areas like coding theory, since they offer the most efficient information transmission [19]. To use this result in the present context however, the 8d spaces of the two E_8 groups have to be treated distinctly, something consistent with the parity violating assumption $g_F >> g_K$ made previously, since couplings emerge from the volumes Vol_c of the compactified dimensions and are inversely proportional to them. Taking the E_8, E_8' to be localized on distinct 8d hypersurfaces (hyps) and the compactification radius corresponding to E_8 to be much smaller than the one of E_8', i.e. $\mathrm{Vol}_c(E_8 \text{ hyps}) << \mathrm{Vol}_c(E_8' \text{ hyps})$, could possibly give a geometrical interpretation of parity-symmetry breaking. Treating the two E_8s distinctly from this particular viewpoint is anyway required, since, while each of the two E_8 lattices provide the densest sphere packing in 8d, in 16d other lattices like the Barnes-Wall (BW) lattice provide a denser sphere packing than $\Gamma_8 \times \Gamma_8$. However, since lattices like BW do not correspond to any root system, they cannot generate the symmetries needed.

Densest sphere packings in higher dimensions $d > 8$ are most likely either disordered, not corresponding to lattices, or the lattices they correspond to are not associated with root systems and thus known gauge symmetries. The disordered packing phenomenon appears already when $d = 9$ and it is due to the fact that the packing density ϕ, defined as the ratio of volume of one sphere to the volume of the corresponding lattice's fundamental cell, is falling exponentially with d. The E_8 lattice Γ_8 for instance provides a packing density of only $\phi(\Gamma_8) = \pi^4/384 \sim 25\%$ in eight dimensions, while in three dimensions, the highest sphere packing density, in an arrangement correctly conjectured by Kepler and being part of Hilbert's 18th problem, reaches around 74%. However, even when such higher-dimensional densest sphere packings correspond to lattices, like the "Leech" lattice in 24d, these do not correspond to a group's root system. Therefore, all other cases for $d > 8$ cannot lead to the symmetries needed for phenomenology and unification considerations.

An additional argument supporting this scenario is related to the concept of optimal lattices. In order for $\Gamma_8 \times \Gamma_8$ to arise naturally, it could be shown that it either extremizes an effective "potential" between lattice points, or equivalently nature is based on a new fundamental principle reformulating the "least-action" principle, requiring "most efficient information transmission" for each of the 2 lattices of the E_8 groups, justifying thus their choice as the vacuum of our world. This lattice would then have to be universally optimal, independently of the specific form of the "potential" used, apart from general requirements of being repulsive at short- and attractive at long distances. Note that universal optimality of this lattice has already been investigated and is true for potentials, as functions of the distance between the centres of the spheres located at the origin of the roots, which are decreasing monotonically fast enough [20]. Repulsion at short distances amounts to the impossibility of having 2 lattice sites occupying the same position, which is equivalent to Pauli's exclusion principle. Attraction at large distances leading to the formation of this lattice is achievable *via* dynamics favouring long-range order.

Alternative justification for the number of internal dimensions (14 here) as multiple of seven might come from arguments based again on densest sphere packing and involving optimal vacuum energy density [21]. In any case, all these arguments might be an indication that we are approaching a theory starting from an action as simple as the one in equation 10

and yielding the dimension of the internal space and its isometries by the dynamics of the action itself, without having to postulate them *a priori*. This is similar in spirit with emergent geometry and gravity [22], although it seems more general since it tries to include all the gauge symmetries as well. The next section deals with this issue in more detail.

3. Critical behaviour and emergence of symmetry

In order to connect our action to the results above, one needs first to justify the value of $\alpha(\Lambda_{GUT})$ calculated in section 2. We are dealing with a critical phenomenon breaking $E_8 \times E_8'$ and having as order parameters the non-zero vevs in equation 8. The relevant critical parameters are the couplings of the two E_8s. The order parameters are assumed here to scale as $E_\mu^m \sim \frac{p^m p_\mu}{M_{\text{Planck}}} < \bar{\Psi}\Psi >$ with fermions Ψ in the fundamental reps of E_8 and E_8' and $|p^m|, |p_\mu| \sim M_{\text{Planck}}$. The $< \bar{\Psi}\Psi >$ condensate, corresponding to the MAC, is assumed to be the catalyst for the formation of the antisymmetric condensates in the $\mathbf{248}_a$ of the two E_8s. Similarly, it is taken to be the catalyst for the formation of the 4-fermion operators discussed in section 2. This should obviate problems arising from the fact that the antisymmetric channel is by itself not as attractive as the singlet one. This implies that the values of the critical couplings for the formation of such vevs are at least close to each other.

Since $E_8 \times E_8'$ breaks at around M_{Planck}, we first estimate this coupling by using the Nambu-Jona-Lasinio (NJL) formalism, which in 4d gives

$$\frac{< \bar{\Psi}\Psi >}{M_{\text{Planck}}^2} \equiv m = \frac{\lambda}{M_{\text{Planck}}^2} \int_0^{M_{\text{Planck}}^2} \tilde{k}^2 d\tilde{k}^2 \frac{m}{\tilde{k}^2 + m^2} \tag{12}$$

with fermion mass m and an effective coupling λ given by

$$\lambda = \left(1 - \frac{m^2}{M_{\text{Planck}}^2} \ln\left(1 + \frac{M_{\text{Planck}}^2}{m^2}\right)\right)^{-1} \tag{13}$$

and having a critical value $\lambda_c = 1$ (see for instance [23]). Assuming that λ is determined by the value a_{SB} of the gauge coupling at the symmetry breaking scale, we find $\lambda \sim \frac{3\alpha_{SB}}{4\pi} C_2$ with $C_2 = 30$ the quadratic Casimir invariant of E_8 [24]. A value $m \sim M_{\text{Planck}}/10$ yields $\lambda \sim 1.05 > \lambda_c$ and $\alpha_{SB} \sim 0.15$. Although this number is small due to the magnitude of C_2, it is larger than $\alpha(\Lambda_{GUT}) = 0.029$ found in section 2. In fact, $\alpha(\Lambda_{GUT})$ is close to its critical value [23] in the case of unbroken E_8 gauge symmetries. This may be due to the fact that gravitational interactions stemming from $LG \subset E_8 \times E_8'$ are associated with massless Goldstone bosons identified with gravitons, and are also connected to E_μ^m which have infinite correlation length. This implies a graviton propagator whose scalar part for small momenta k scales as $1/k^2$ [25]. Since in a unified setting the gravitational constant scales as $G \sim \lambda/M_{\text{Planck}}^2$, neglecting this infrared (IR) behaviour amounts to neglecting the masslessness of gravitons. The enhanced contribution to the condensate integral stemming from the IR region might be responsible for the smaller critical value of λ.

Although a full calculation requires a complete quantum theory of gravity, we estimate roughly this critical coupling by replacing $1/M_{\text{Planck}}^2$ above with a gauge-boson propagator

$1/(k-\tilde{k})^2$, k being the external momentum. After angular integration this expression is replaced by $1/max(k^2,\tilde{k}^2)$. Taking m and λ to be momentum-independent, we have

$$\frac{<\bar{\Psi}\Psi>}{M^2_{\text{Planck}}} \equiv m \sim \lambda \int_0^{M^2_{\text{Planck}}} \frac{\tilde{k}^2 d\tilde{k}^2}{max(k^2,\tilde{k}^2)} \frac{m}{\tilde{k}^2+m^2} \tag{14}$$

yielding $\lambda = \left(1+\ln(\rho^2+1)-(1+m^2/k^2)\ln\left(1+k^2/m^2\right)\right)^{-1}$, where $\rho \equiv M_{\text{Planck}}/m$. Naturalness arguments lead us to consider $1 \leq \rho \leq 10$. The regime $k \gg m$ is then equivalent to $k \sim M_{\text{Planck}}$, in which case λ is given by the same expression as in equation 13, so $\lambda_c = 1$. For $k \sim m$ however, $\lambda \sim 1/\left(1+\ln(\frac{\rho^2+1}{4})\right)$. Taking $\rho \sim 10$ as before, we have $\lambda \sim 0.24$. Then, we find $\alpha_{SB} \sim C_2^{-1} \sim 0.03 \sim \alpha(\Lambda_{GUT})$, allowing thus a dynamical interpretation of $\alpha(\Lambda_{GUT})$ found in the previous section. Similar results are obtained for $k \ll m$. Criticality is not apparent for $k \leq m$ and $\rho \sim 10$, but α is close to $\alpha_c = \frac{\pi}{3}C_2^{-1} \sim 0.035$ predicted for self-energies $m = m(k)$ in unbroken gauge theories [23]. With regards to equation 5, the wide hierarchy between M_{Planck} and the weak scale can then be traced back to the magnitude of C_2, since

$$\Lambda_K \sim M_{\text{Planck}} \exp(-1.23\, C_2). \tag{15}$$

The coupling in higher dimensions being inversely proportional to Vol_c, a shrinking compactification space until the above equation is satisfied might provide a relevant geometrical interpretation.

As promised in the previous section, we now take a first glance at the dynamics which might lead to the emergence of symmetry in the first place by studying a relevant phase transition. Deferring rigorous justification of this approach to a more detailed future study, we apply techniques borrowed from similar studies in solid-state physics, chemistry, biology and even sociology. The common starting ground is the emergence of particle configurations exhibiting spontaneous self-organization transitions in ordered structures and nucleation, like DNA or crystals, *i.e.* processes characterized by "self-organised criticality" [26]. For instance, a numerical study of a liquid-to-crystal freezing transition of hard spheres shows that the emerging crystal, after the liquid has been subject "slowly" to pressure, minimizes its potential by a densest-sphere packing arrangement [27]. Faster compression rates lead to crystals containing defects and to "liquid-to-glass" transitions, corresponding here to a cosmological scenario lacking the symmetries needed, since glass corresponds to a disordered phase. According to these studies, effective-potential minimization on the emerging lattices is carried over to the dual lattices, in a way that long-range, non-local effects in one lattice can be substituted by local effects in its dual. Therefore, the self-duality property of the $E_8 \times E_8'$ lattice mentioned previously might provide a further advantage of the scenario proposed here. First, we find the relevant universality class, allowing us to predict the qualitative behaviour of the system by studying a simpler model Hamiltonian belonging to the same class. One crucial factor determining universality class is dimensionality, noting that 8, the dimension of the E_8 lattice, is the upper critical dimension for several random physical systems, generic lattice trees and some polymer and percolation models [28], appearing to be critical also for transitions in glasses [29] and allowing the application of mean-field theory results.

We place our action on a lattice to see if its qualitative behaviour can be inferred by simpler or similar systems in lattice gauge theories or solid-state systems. The effective action S_{lat} stemming from equation 9 to lowest order, apart from the Einstein-Hilbert terms, is written as $S_{lat} = \sum_{<i,j>} \mathscr{E}_{ij} \bar{\Psi}_i \Psi_j$, where $\mathscr{E}_{ij}$ is an antisymmetric matrix proportional to the system's volume and encoding information on E^m_μ. The fact that the sum over the lattice sites i, j is restricted over nearest neighbours, denoted by $< i, j >$, originates from the partial derivative in the action. Gauge fields and spacetime do not appear yet, our goal being to have them emerge rather than postulate their appearance *a priori*.

The form of S_{lat} is reminiscent of the Edwards-Anderson (EA) spin-glass Hamiltonian with zero external field, given by $H_{EA} = -\sum_{<i,j>} J_{ij} S_i S_j$ [30], with locally-interacting spin fields $S_{i,j}$ taking values ± 1 and J_{ij} random variables obeying a zero-mean distribution function. The minus sign in front of H_{EA} makes the EA model favour long-range, ferromagnet-like, order at low temperatures T. To render this compatible with S_{lat}, we treat the fermion kinetic term as corresponding effectively to an interaction. A problem arises from the fact that the $S_{i,j}$ are bosonic while the $\Psi_{i,j}$ are fermionic fields. We assume in the following that the two models above belong to the same universality class in order to draw some conclusions on their qualitative behaviour which should not be influenced by field statistics, at least for large T. The anticommutativity of fermions, an expression of Pauli's exclusion principle, guarantees that these lie on distinct locations, something which in bosonic Ising- or Potts-inspired models is ensured by assuming *a priori* an underlying lattice. Therefore, positing the existence of a lattice approximates one feature of Fermi statistics.

The universality class of similar models depends on the average coordination number c, ı.e. the number of nearest neighbours of each lattice site. An 8d square lattice $\mathscr{Z}^8$ with $c = 16$ gives a different behaviour from the E_8 lattice having $c = 240$, even though they are both defined in 8d. Models with large c, such as the one defined on a E_8 lattice, exhibit dynamics described by the mean-field approximation [31][32]. The EA model approaches thus models on random Bethe lattices and complex network theory studying random structures. These exhibit topological phase transitions when the bond concentration probability p exceeds a critical value p_c called "percolation threshold", which for large c is given by $p_c \sim 1/c$. For $p < p_c$, only finite clusters of edges connecting lattice sites appear, while for $p > p_c$ the whole lattice is occupied by a huge cluster [32]. This proves to be crucial for the discussion below.

In order to study the emergence of the E_8 lattices from first principles, we use a model for percolation phenomena assumed here to belong to the same universality class, *i.e.* the single-state ($q = 1$) Potts model, with Hamiltonian H_P, similar to H_{EA}, given by $H_P = -J \sum_{<i,j>} \delta(S_i, S_j)$, where $J > 0$ is the coupling strength and $\delta(S_i, S_j) = 1$ when $S_i = S_j = 1$ and zero otherwise. The partition function $Z = \sum_{C_i} e^{-\beta H_P}$ is given by

$$Z = \sum_{C_i} Z_i = \sum_{C_i} \left(e^{\beta J} - 1 \right)^{E_i} \tag{16}$$

where the sum is over clusters C_i consisting of E_i edges and $1/\beta = k_B T$. Here, p is given by $p = 1 - e^{-\beta J}$ and increases with decreasing T. This model exhibits a 2nd-order phase transition for low T, which for large c implies a mean-field behaviour for the 2-point

correlation function given by $<\tilde{S}_i\tilde{S}_{i+k}>\sim 1/k^2$. Such discrete models are usually studied on lattices with given dimensionality d and c. In the following, we explore the behaviour of a system of nodes minimizing its free energy by adjusting its d and c in order to form an optimal lattice.

For high T, $w \equiv e^{\beta J} - 1 \sim p \sim \beta J \ll 1$ in equation 16 implies that only clusters with few edges contribute significantly to the partition function. In a competition between annihilation and aggregation of large clusters, those of low d and c dominate. Filaments or low-dimensional surfaces formed by edges connected with each other are expected to form topological entities resembling "time" with 1 or 2 "space" dimensions, without more structure. It is worth reminding however that such configurations are not accurately described by mean-field theory. In this high-T regime, each cluster C_i consisting of E_i edges gives a positive contribution to the system's free energy $F_{C_i} = -\ln Z_i/\beta \sim -E_i \ln(\beta J)/\beta$. The positive free energy, compensated by the system's gradual cooling, is identified with a vacuum energy, *i.e.* with Λ. It is accompanied, for a system of volume Vol, by a negative pressure $P = -F_{C_i}/(Vol)$ leading to expansion and probably to an inflationary scenario for the early Universe. The same cluster C_i gives a negative contribution $S_{C_i} = -\partial F_{C_i}/\partial T \sim k_B E_i \Big(\ln(\beta J) - 1\Big)$ to the system's entropy S, rendering the formation of large clusters of edges very costly, energetically and entropy-wise. This system of nodes, a rough model of the "pre bib-bang" world, lies initially in a highly-probable state, ı.e. having large T and S, possibly obviating the need for contrived cosmological boundary conditions. Evolution is dictated by the system's need to reduce its energy, which is achieved by lowering T and expanding. This might define in parallel an "arrow of time", the increase in S compensating the entropy loss due to the formation of the spacetime "crystal", possibly providing a hint towards an explanation of the 2nd law of thermodynamics.

There is a certain T however for which the behaviour of the partition function Z in equation 16 changes dramatically. This change proves to be crucial for our argument towards symmetry emergence. For low T such that

$$k_B T \leq k_B T_c = J/\ln 2 \sim 1.4J \tag{17}$$

we find $p \geq 1/2$ and $w \geq 1$, implying that large clusters consisting of many edges dominate over the smaller ones! Readers familiar with the q-state Potts model recognize in this expression T_c given by $e^{\beta_c J} = (1+\sqrt{q})/\sqrt{q}$ for $q = 1$. This has a far-reaching impact on the topology of the network of nodes. Since for a network to have any sense we assume that $\min(E_i) = c$, a lattice like the one of E_8 with $c = 240$ contributes much more to the partition function than the conventional $\mathscr{Z}^8$ lattice having $c = 16$. Approaching T_c amounts therefore to having $E_8 \times E_8'$ emerge spontaneously from the dynamics. The relevance of densest-sphere packing arguments presented in the previous section is now apparent, since this particular lattice offers an optimal configuration with regards to c and might be preferred over alternative arrangements not offering so many edges per node. Note moreover that "crystal" clusters with $E_i = 240$ evolve even when $T > T_c$. However, such clusters should lie within limited regions not contradicting Big-Bang nucleosynthesis.

At this critical point other lattices with even higher d and c could also form. These

however do not lead to the symmetries observed in our world, as explained in the last section. This implies that we might be living within a metastable region, with other Universe domains corresponding to different configurations of lattice points, devoid of the known interactions. This is consistent with Ostwald's rule in polymorphic and allotropic crystallography, according to which the least stable polymorphs crystallize first, leading to transformations between closest phases with regards to free energy. Metastability of the two E_8 lattices is central to the argument presented, should be studied more thoroughly, and is at least qualitatively supported from the fact that these lattices offer a local maximum for the centre density, defined as the ratio of sphere-packing density over the unit-sphere volume for certain dimensions d. Indeed, there exist bounds according to which the E_8 lattice offers a centre density higher than the maximum achievable for $8 < d < 12$ [33]. In parallel, regarding known sphere packings, it offers maximal centre density for $6 < d < 18$. The next dimension probably providing a local maximum for the centre density hosts the even unimodular Leech lattice in $d = 24$ offering maximal centre density for $0 < d < 28$. Although it exhibits symmetries not associated with the known Lie groups, one should explore further the relevant Physics since it might correspond to a more stable equilibrium where our Universe might eventually decay into.

Returning to the E_8 lattices now, near $T = T_c$ one finds a free energy $F_{C_i} \sim k_B(T - T_c)\ln\Omega_{C_i}$ and an entropy $S_{C_i} \sim -k_B \ln\Omega_{C_i}$, where $\Omega_{C_i} = 2^{2E_i}$ has a combinatorial interpretation, expressing the number of classically distinct configurations depending on whether a pair of nodes is connected or not by an edge. Since for $T \equiv T_{c+}$ just above T_c one expects $E_i \sim 1$, while for $T \equiv T_{c-}$ just below T_c we have $E_i \sim 240$, the entropy $S(T)$ is discontinuous and the heat capacity $C \equiv T\partial S/\partial T$ is expected to diverge at $T = T_c$. Moreover, at $T = T_c$ we expect a latent heat, or enthalpy, equal to

$$H = T_c\big(S(T_{c+}) - S(T_{c-})\big) = 478J. \tag{18}$$

Another regime is $k_B T \ll J$, for which $F_{C_i} \sim -E_i J < 0$ and $\mathrm{S}_{\mathrm{C_i}} \sim 0$. This describes the ground state and has a clear intuitive interpretation favouring large clusters. Zero entropy follows the 3rd law of thermodynamics. Negative energy implies positive pressure and contraction. However, quantum corrections are here significant. The negative contribution stemming from cluster formation is nearly cancelled by the quantum mechanical ground-state energy, leading to roughly zero energy and pressure, and lattice points close to equilibrium. A more detailed study of critical behaviour in this context clearly necessitates computer simulations, which have proven to be indispensable even for much simpler physical systems. In particular, it would be very useful to further study the metastability of the various possible lattices, in order to check the validity of the emergence scenario presented. For temperatures close to criticality, it might prove to be useful to also study the behaviour of the $q = 2$ Potts model as being closer to the fermionic action we started with.

Next, we describe some potentially interesting cosmological implications of this critical behaviour. We take the measured $\Lambda \sim 2 \times 10^{-3}$ eV to correspond to the free energy of a minimal cluster of the $E_8 \times E_8'$ lattice having 240 edges from each of the two E_8's, *i.e.* $E_i = 480$, and $T_{CBR} \sim 2.7 \times 10^{-4}$ eV of the cosmic background radiation to be equal to the

system's temperature. Then, the critical free-energy expression yields $\Lambda \sim 7k_B T_{CBR} \sim 10J$ and $\varepsilon \equiv (T_{CBR} - T_c)/T_c \sim 1\%$. This implies a certain fine-tuning close to criticality for T. Taking the Universe to be in a "glass-to-crystal" transition, typical relaxation and equilibration times τ for glassy dynamics are huge compared to the microscopic ones of ferromagnetic-type systems. This leads to considering non-adiabatic phenomena, since τ is given by $\tau \sim \xi^z \sim (T - T_c)^{-\nu z}$, with ξ the correlation length and ν, z critical exponents. Near the transition point, ξ and τ diverge. This implies that relaxation times are of cosmological scale, and we might be just living within such a critical period.

Another issue to address is the smallness of Λ in comparison to M_{Planck}. The solution to this puzzle might be coming from the Hamiltonian we started with, which includes only local, nearest-neighbour, interactions, effectively introducing a very large infrared cut-off. For this interpretation to work, Feynman integrals are to be performed over the whole momentum space only when particles are present in the Feynman diagram, which can exhibit non-local behaviour. Spacetime itself is local and momentum integration should not be allowed to reach values much lower than M_{Planck}. Alternatively, one can introduce a phenomenological potential $V(r)$ between lattice sites being separated from each other by a distance r, similar to the Lennard-Jones type, given by

$$V(r) = -2E_i J \left(\frac{L_{\text{Planck}}}{r}\right)^{d-3} \left(1 - \frac{\sqrt{Ja}}{2}\left(\frac{a}{r}\right)^d\right) \tag{19}$$

with $L_{\text{Planck}} = 1/M_{\text{Planck}}$, $d = 16$ and a having dimensions of length corresponding to the distance where "repulsive" effects become important. For large r, $V(r)$ vanishes like $1/r^{d-3}$, since the 2-point correlation function in position space falls as $1/r^{d-2}$. Using the fact that $d \gg 1$, $\min(V(r)) \sim -E_i J$ in accordance with the ground-state free energy reached for $r_{min} \equiv L_{\text{Planck}} \sim a(Ja)^{1/2d}$ defining in parallel L_{Planck} as a function of J. The potential exhibits a non-linear behaviour with respect to J, a situation possibly traceable back to the action of equation 10 before linearization. Moreover, $V(r)$ has a zero at $r = 2^{-1/d} L_{\text{Planck}} \sim L_{\text{Planck}}$, i.e close to the value where it has its minimum.

Since the slope of $V(r)$ for small r is proportional to a large power of a, adjusting the modulus a can control the steepness of the potential. Using

$$JL_{\text{Planck}} = \left(\frac{L_{\text{Planck}}}{a}\right)^{2d+1} \tag{20}$$

and taking $a \sim 9.3 L_{\text{Planck}}$ gives the required hierarchy $\Lambda \sim 10^{-31} M_{\text{Planck}}$ between Λ and M_{Planck}. The extent of this hierarchy might therefore be traced not only in the large dimensionality of our space, but also in the steepness of the repulsive potential between sites. Moreover, the quantity JL_{Planck}, apart from falling with increasing a, is proportional to the system's Vol_c and inversely proportional to the emergent gauge coupling. Symmetry breaking could then be associated with a shrinking Vol_c and with a critical value of the steepness parameter a, *i.e.* $a_c \sim 9.3 L_{\text{Planck}}$, above which fermion condensates form. Anyway, the form of the 2nd term of the potential needs further justification, probably in terms of a series expansion in powers of $(a/r)^d$, where the value of a might be determined by the geometry of the lattice.

Consequently, a higher-dimensional analogue of spin-glass phase transitions might provide a picture for the emergence of $E_8 \times E_8'$ at the beginning of our Universe, as a kind of "liquid-to-solid", freezing phase transition, or a kind of disorder-order, "glass-to-crystal" transition. Regarding entropy S, the only way for the system to compensate for the loss of S within a spacetime volume Vol during a time dt is to expand, changing its volume by $d(Vol)$. This leads to an equation $Vol = (Vol)_0 \exp(\mu E_i t)$ with μ constant. Although large values for E_i imply a period resembling inflation, a study in this direction exceeds the bounds of the present analysis. Other cosmological implications include the existence of macroscopic domains in the Universe not having the symmetries observed in our neighbourhood. Particles within such regions would not interact in familiar ways, for instance not feeling electromagnetic interactions and possibly supplying an explanation for Dark Matter (DM). The luminous parts of galaxies would occupy regions corresponding to the "jammed", ordered phase of spacetime, domain states of ferromagnetic type, like "crystal bubbles" within a glass-type, amorphous spacetime structure. Domain growth would be described by a relation of the form $\xi(\tau) \sim \tau^{1/z}$. The ratio R of crystal-to-glass-type volumes would be given by $R = 1 - \exp(-\Delta F/k_B T)$, where ΔF is the free energy gained by the system by being in the "crystal" state. Nucleation and growth of crystalline grains within amorphous glass materials is a frequently-studied subject in solid-state physics [34] and could provide a testing ground for related cosmologies.

A related scenario that could be analysed might predict that "spacetime" nucleation continues today, implying a growth of the luminous-to-DM ratio on cosmological time-scales. Using the expression for the critical free-energy, we find $R = 1 - 2^{-2\varepsilon(480-\tilde{E}_i)}$ where $\tilde{E}_i$ is the number of edges, per potential $E_8 \times E_8'$ lattice cluster, of the "glass" state. For $\varepsilon \sim 1\%$, we find the following possible characteristic $(R, \tilde{E}_i)$ pairs: $(5\%, 476)$, $(24\%, 460)$ and $(75\%, 380)$. A detailed analysis towards this direction would allow the prediction of galactic DM concentrations and of the average structure of the underlying spacetime lattice. It would allow answering questions like "is the structure of spacetime within the intergalactic voids of glass- or crystal-type?" and compare results with DM considerations [35]. Assuming that "crystal" domains are occupied by visible galaxies implies that $R \sim 5\%$, while taking intergalactic voids to be also "crystal"-like raises R to around 77%. Another possibility is that DM regions correspond to denser "sphere packings" not linked to a group's root system. This takes us to a scenario where our vacuum has already started decaying towards a more stable configuration like the Leech lattice, leading to growing DM and shrinking luminous domains.

Alternatively, in off-equilibrium phenomena of crystal and glass formation, the fluctuation-dissipation theorem in fast transitions is violated, since the system does not have enough time to relax to its new equilibrium, forcing us to consider "effective temperatures" T_{eff} even an order of magnitude larger from the heat-bath ones [36]. Here, $R \sim 50\%$ implies $T_{eff} \sim 10 T_{CBR}$, while $R \sim 80\%$ implies $T_{eff} \sim 5 T_{CBR}$. If the universe expanded and cooled too fast to have relaxed to equilibrium, T_{eff} for "crystal" formation is larger than T_{CBR}. This might explain expansion or inflation in terms of $\Delta F = \Delta F(t)$. A disordered initial configuration in a "liquid" state has a higher energetic gain by forming a crystal than a "glass" state. As the Universe cools down and ordered "glass-type" structures emerge, $\Delta F(t)$ decreases. Such

values of T_{eff} are consistent with treating particles as topological or vacancy defects on the lattice background described above, their number density d_p in the Universe being a function of enthalpy cost and approximately equal to the ratio of their entropy $S_p \sim k_B 10^{89}$ to the Universe entropy $S_B \sim k_B 10^{122}$ assuming a Bekenstein-bound saturation [37], i.e.

$$d_p \sim S_p/S_B \sim 10^{-33} \sim \exp\left(-H/k_B T_{eff}\right), \tag{21}$$

implying from equation 18 that $T_{eff} \sim 4.4 T_{CBR}$. This result favours in parallel the characterization of intergalactic voids as “crystal-like”.

Moreover, the position of galaxies and such “crystal bubbles” might be correlated, with mass acting as a topological defect closely connected to the formation of a spacetime “crystal”. This has far-reaching implications on the structure formation of galaxies, consistent with the view that stars are born within DM halos. It could potentially lead perhaps to an understanding of the shape of spiral galaxies on the basis of “helicoidal dislocations” in crystals. It might explain the large voids between galactic clusters, since crystals usually displace impurities towards boundaries of different phases and form vacancy clusters to minimize their energy. It might also solve the “dwarf galaxy” problem, i.e. the rarity of “dwarf” galaxies, which are an order of magnitude less than predicted by simulations [38][39], since in these “dwarf” galaxies, particle density, seen as a defect, has not reached values consistent with nucleation and “crystal” formation. Experiments could be designed in the far future to probe the spacetime structure within DM domains, or to measure the potential energy release when the “crystal” forms. However, critical phenomena of this kind are intractable even in low dimensions, necessitating the use of phenomenological potentials and simulations in the area of glass-to-crystal transitions which is still far from being well understood. Although a thorough analysis in this direction transcends our purposes, the smallness of the luminous-matter portion of the total energy of the Universe makes us reluctant to discard so radical solutions of the DM and Dark-Energy puzzle too hastily. We close this discussion by noting a similar attempt to circumvent problems cold-DM models have with galactic mass distribution by positing that DM is topological [40].

Next, we discuss briefly quantization of our action. What follows is speculative, noting a proximity of the present theory with some current approaches to quantum gravity. The fact that the group E_8 is simply laced allows the definition of a common scale $1/M_{\text{Planck}}$ equal to the roots norm, an identification defining a fundamental scale for the theory. By having $\Gamma_8 \times \Gamma_8'$ emerge with lattice spacing equal to L_{Planck}, one achieves a cellular decomposition of spacetime with a UV cut-off equal to M_{Planck}, possibly avoiding in principle singularities plaguing quantum gravity. The number of faces, edges and vertices of the various simplices is determined by this lattice. This might have a dramatic impact not only on the general renormalization programme but also on black holes, the initial singularity of spacetime and gravitational collapse, analogous for some to the false prediction of atom collapse before the advent of quantum mechanics.

Moreover, the metric in section 2 is reminiscent of the one in the spinorial version of Ashtekar variables [41] using no background metric. In an approach close to spin networks and lattice Yang-Mills, we have lattice nodes corresponding to 4d spacetime points, a “world

crystal", where a continuum perturbative limit is ill-defined since spacetime is inherently discrete, while flat Minkowski space lies far from perturbative considerations. On each node there is a fiber corresponding to $E_7 \times E_7'$ stemming from 14 compactified dimensions. A probability wavefunction extending between 2 adjacent lattice nodes corresponds to a particle with Planck-scale energy, which seems a reasonable configuration for the first moments after the creation of the Universe. It also provides an understanding of Heisenberg's uncertainty principle, since a particle is not localized on a node but extends at least between 2 adjacent nodes. This is compatible with treating particles with spin as spacetime defects like "dislocations" and "disclinations" in Einstein-Cartan action theories [42][43], reminds us of Wheeler considering particles as "quantum geometrodynamic excitons" in an analogy inspired by solid-state Physics and is consistent with our previous treatment of particles as topological or vacancy defects. Such schemes might also lead to a small Λ due to a kind of "equilibrium" between lattice sites [43]. The above provide just a heuristic hint towards quantization, more work being required to derive robust results. Experiments around M_{Planck} should distinguish such a spacetime fabric from models treating particles as extended objects on a continuous spacetime background. Ideas along these lines might include in the future a gravitational analogue of Bragg spectroscopy probing the microstructure of spacetime.

4. Discussion

We exposed above an attempt to unify gauge with gravitational interactions using $E_8 \times E_8'$ emerging from first principles. It presents several advantages, not requiring many arbitrary parameters, nor fundamental scalar fields, nor extra space dimensions; it leads to coupling unification and to an understanding of the unification coupling strength from a group invariant; it tries to reproduce the symmetries, the family structure of matter and the dimensionality of spacetime, far from treating it as background; it provides a possible solution to the hierarchy problem between the Planck scale, the weak scale and the cosmological constant scale. Moreover, it exhibits a unique vacua sequence with cosmological implications like the interpretation of DM as having a topological origin. Securing the present approach on a firm basis needs, among many other things, a new physical principle, more fundamental than a given spacetime or gauge symmetry; according to it, spacetime, matter, and their symmetries, emerge naturally, after a relevant phase transition, from a set of identical anticommuting fields connected to each other optimally.

However, several open issues still remain. Most important, the issue of reproducing correctly the equations of General Relativity starting from an action of the form given in equation 10 and the issue of providing a solid framework leading to its quantization, while the emergence of a UV cutoff in the theory is an encouraging sign in this direction. Next, one should show that the 16d torus over the $\pi\Gamma_8 \times \pi\Gamma_8$ lattice emerges naturally and produces the action needed to describe correctly Physics at long wavelenghts, including the number of internal dimensions, their possible relation to inflation, and the value of the cosmological constant. In this respect, the metastability of this lattice should be studied further, since it would offer a solid argument for the emergence of the symmetries observed in nature. In

addition, more work is required towards proving that the Potts lattice model analyzed belongs to the same universality class as the one required by our spinor-gravity approach, in order to draw safer conclusions on the validity of the qualitative cosmological implications of this model. Then, a justification of the parity-violating assumption $g_F >> g_K$ is needed, probably in relation with the compactification volumes. Furthermore, a rigorous justification of the breaking channels of the symmetries down to the SM should be sought, and a more reliable calculation of the critical coupling associated to the breaking of the initial symmetry should be performed. Moreover, unitarity issues have to be tackled because of the non-compact form of the Lorentz group $SO(3,1)$. In principle, a metric-independent topological action like the one in equation 10 should cure such issues when considered from the viewpoint of a fundamental theory.

On the experimental front, a problem with the fermion content described in this work is that it is usually associated with an S parameter which is larger than what is measured experimentally. It remains to be seen whether non-perturbative vertex corrections in such models can drive the S parameter close to zero, although so many new fermions are introduced [6]. On the other hand, LEP and Fermilab results at the $2-3\sigma$ level suggest mass-dependent anomalous quark couplings [44] and an excess of dijet plus W-boson events [45], signals which are in principle compatible with quark-katoptron mixing and the existence of katoptron bound states [46]. We expect future collider experiments in LHC and elsewhere to shed more light on some of these issues.

Acknowledgments

This work has benefited from discussions with T. Appelquist.

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